

Find the area bounded by the curves $y = 4x^2 + 8x$ and $y = 4x^3$.

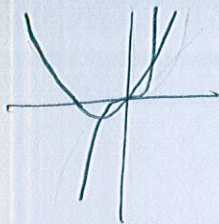
NOTE: Your final answer must be a number, not an integral nor sum of integrals.

$$4x^2 + 8x = 4x^3$$

$$0 = 4x^3 - 4x^2 - 8x$$

$$0 = 4x(x-2)(x+1)$$

$$x = -1, 0, 2$$



SCORE: ____ / 8 PTS

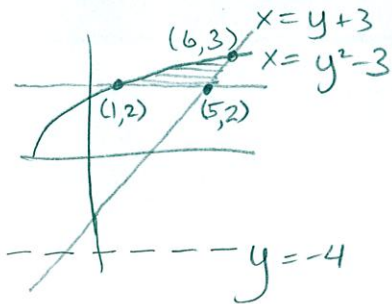
① EACH

$$\begin{aligned} & \int_{-1}^0 (4x^3 - (4x^2 + 8x)) dx + \int_0^2 (4x^2 + 8x - 4x^3) dx \\ &= \left(x^4 - \frac{4}{3}x^3 - 4x^2 \right) \Big|_{-1}^0 + \left(-x^4 + \frac{4}{3}x^3 + 4x^2 \right) \Big|_0^2 \\ &= 0 - \left(1 + \frac{4}{3} - 4 \right) + \left(-16 + \frac{32}{3} + 16 \right) \\ &= \frac{37}{3} \end{aligned}$$

The region bounded by $y = \sqrt{x+3}$, $y = x-3$ and $y = 2$ is revolved around the line $y = -4$.

SCORE: ____ / 8 PTS

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.



$$y+3 = y^2-3$$

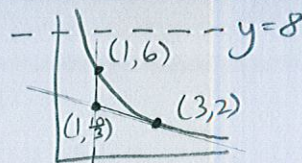
$$0 = y^2 - y - 6 \quad \textcircled{1}$$

$$0 = (y-3)(y+2)$$

$$y = 3, -2$$

$$\underbrace{2\pi}_{\textcircled{2}} \int_{\textcircled{2} 2}^{\textcircled{1} 3} \underbrace{(y+4)}_{\textcircled{2}} \underbrace{(y+3-(y^2-3))}_{\textcircled{2}} dy$$

Consider the region defined by $y \leq \frac{6}{x}$, $y \geq 4 - \frac{2}{3}x$ and $x \geq 1$.



SCORE: ____ / 14 PTS

[a] If the region is revolved around the line $y = 8$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

$$\begin{aligned} \frac{6}{x} &= 4 - \frac{2}{3}x \\ 18 &= 12x - 2x^2 \\ 2x^2 - 12x + 18 &= 0 \\ \frac{2(x-3)^2}{2} &= 0 \\ x &= 3 \end{aligned}$$

[i] using the disk or washer method (**NOTE: You do NOT need to simplify your integrand.**)

$$\frac{1}{2} \pi \int_1^3 \left(\left(8 - \left(4 - \frac{2}{3}x \right) \right)^2 - \left(8 - \frac{6}{x} \right)^2 \right) dx$$



EXPLANATION: ① FOR R
① FOR r
① FOR $R^2 - r^2$

① FOR EACH ITEM UNLESS OTHERWISE NOTED

[ii] using the shell method (**NOTE: You do NOT need to simplify your integrand.**)

$$2\pi \left(\int_{\frac{10}{3}}^6 (8-y) \left(\frac{6}{y} - \frac{2}{3}(4-y) \right) dy + \int_{\frac{10}{3}}^6 (8-y) \left(\frac{6}{y} - 1 \right) dy \right)$$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the x -axis are isosceles right triangles with their hypotenuse in the base. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$$\frac{1}{4} \int_1^3 \left(\frac{6}{x} - \left(4 - \frac{2}{3}x \right) \right)^2 dx$$

